GENERICITY AND SMALL SETS IN ANALYSIS

May 26-28, 2015

"Domaine du Rond-Chêne"

Abstracts

Short courses

- Frédéric BAYART (Clermont-Ferrand):
  Small and big sets in analysis: genericity, prevalence and beyond

  In this short course, we will survey several notions of negligible sets in analysis (sigma-porous sets, Haar null sets, Gauss null sets...). We will study the links between these notions as well as examples where they come into play.

- Juan B. SEOANE SEPULVEDA (U. Compl. Madrid) :
  Lineability: the search for linearity in mathematics

  For the last decade there has been a generalized trend in Mathematics on the search for *large* algebraic structures (linear spaces, closed subspaces, or infinitely generated algebras) composed of mathematical objects enjoying certain *special* properties.

  This trend has caught the eye of many researchers and has also had a remarkable influence in Real and Complex Analysis, Operator Theory, Summability Theory, Polynomials in Banach spaces, Hypercyclicity and Chaos, general Functional Analysis, and even in Probability Theory.

  Throughout the lectures within this mini course we shall provide an account on the advances and on the state of the art of this trend, nowadays known as *lineability* and *spaceability*, within the framework of Real and Complex Analysis.

Lectures

- Richard ARON (Kent State University):
  Subspaces of the zero set of a polynomial

  Let $P : X \to \mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ be an $n-$homogeneous polynomial, where X is a real or complex Banach space of finite or infinite dimension. In this expository talk, we discuss various positive and negative results concerning the lineability of $P^{-1}(0)$. Among topics that we hope to consider are maximal subspaces contained in the zero set of P. Our results depend on the dimension and geometry of X, on the homogeneity n of P, and of course on the scalar field.

- Zoltan BUCZOLIC (Budapest):
  Genericity in (sub)spaces of continuous functions

  Typical/Generic properties of continuous functions are always interesting questions in Real Analysis.
  • The domain of these functions can be [0, 1], or in more general setting a higher dimensional interval, or a fractal.
  • The properties we are interested in can be the micro tangent set system, the structure of level sets, or the multifractal structure of local H?older exponents.
  • The "subspace"considered can be the whole space, the space of monotone continuous functions, or the convex functions.

  I obtained various results about generic properties, some of them with various coauthors: R. Balka, U.B. Darji, M. Elekes, A. Máthé, J. Nagy, Cs. Ráti, and S. Seuret. In this talk I want to discuss some of my favorite results in this area.

- Robert DEVILLE (Bordeaux):
  A characterization of the Radon-Nikodym property with an application to the gradient problem

  Following the pionering work of Maly and Zeleny, we obtained, in collaboration with O. Madiedo, the following characterization of the Radon-Nikodym property.
  Theorem : If X is a Banach space, the following are equivalent :
  (1) X has the Radon-Nikodym property.
  (2) For all $f \in S_{X^*}$ and all $\varepsilon > 0$, there exists $t : X \to S_{X^*} \cap B(f, \varepsilon)$ such that for all sequence (x_n) in X, if $(f(x_n) - \varepsilon\|x_n\|)$ is bounded below and $\langle\, t(x_n), x_{n+1} - x_n \,\rangle \leq 0$ for all n, then (x_n) converges.
  This result can be used to give a simple construction of a differentiable function u on R^n, non identically equal to zero with bounded support, and such that $u'(x) = 0$ almost everywhere (the existence of such a function is due to Z. Buckzolich).

- Céline ESSER (Liège):
  Dense-lineability in classes of differentiable functions

  The Denjoy-Carleman classes are spaces of smooth functions which satisfy growth conditions on their derivatives defined through weight sequences. In this talk, given a Denjoy-Carleman class E of Beurling type that strictly contains another non-quasianalytic class F of Roumieu type, we handle the question of knowing how large the set of functions in E that are nowhere in the class

F is. In particular, we prove the dense-lineability of the set of functions of E which are nowhere in F. Consequences for the Gevrey classes are also given. We extend then these results to the case of classes of ultradifferentiable functions defined imposing conditions on their Fourier Laplace transform using weight functions.

- Gustavo MUNOZ FERNANDEZ (U. Compl. Madrid):
  Lineability problems motivated by a classical characterization of continuity
  Joint work with José Luis Gámez, Daniel Pellegrino and Juan B. Seoane-Sepúlveda

  It is well known that a continuous function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets. These two results are a part of any standard course on elementary real analysis in one variable. It is equally easy, but not as well known, that if a function $f : \mathbb{R} \to \mathbb{R}$ transforms compact sets into compact sets and connected sets into connected sets, then f is necessarily continuous. In this talk we will show what happens when we drop only one of the previous two conditions. If, for instance, we drop the compacity condition, a good example of a Darboux function that is not continuous could be found among the discontinuous derivatives. We will provide several examples of derivatives with "many" discontinuities and an optimal result on the lineability of the set of derivatives that are discontinuous almost everywhere. Also, another optimal lineability result will be given about the set of compact functions that are everywhere discontinuous.

  To finish, if we restrict attention to polynomials on an infinite dimensional space, we will see that compacity implies continuity, although it is not clear whether Darboux polynomials are also continuous. In any case, we will provide optimal lineability results on the set of unbounded polynomials.

- Yanick HEURTEAUX (Clermont-Ferrand):
  Generic boundary behaviour for harmonic functions in the ball

  It is well known that if h is a nonnegative harmonic function in the ball of $\mathbb{R}^{d+1}$ or if h is harmonic in the ball with integrable boundary value, then the radial limit of h exists at almost every point of the boundary. In this talk we are interested in the exceptional points and in the speed of divergence at these points. In particular, we prove that for generic harmonic functions in the ball, and for any $\beta \in [0, d]$, the Hausdorff dimension of the set of points ξ on the sphere such that $h(r\xi)$ looks like $(1-r)^{-\beta}$ is equal to $d - \beta$. This is a joint work with Frédéric Bayart.

- Olga MALEVA (Birmingham):
  Lowest fractal dimensions for universal differentiability

  In a given space X, we are looking for as small as possible universal differentiability sets (UDS) S, defined by the requirement that every Lipschitz function on X has a point of differentiability in S. We show that Euclidean spaces contain fractal universal differentiability sets of Minkowski dimension 1. This is the lowest possible as all projections of the set of differentiability points inside UDS have positive measure. We discuss finding the best way to describe how small universal differentiability sets could be. This is joint work with M.Dymond.

- Etienne MATHERON(Lille-Artois):
  When is a positive cone Haar-null?

  The talk will be centred on the following question : if X is a real Banach space with an unconditional basis, when is the associated positive cone Haar null in X?

- Victor SANCHEZ (U. Compl. Madrid):
  Spaceability and operator ideals

  Let I_1 and I_2 be arbitrary operator ideals in the sense of Pietsch and E and F be Banach spaces such that the set $I_1(E, F) \setminus I_2(E, F)$ is non-empty. A quite general sufficient condition on the Banach spaces in order to obtain the spaceability of $I_1(E, F) \setminus I_2(E, F))$ will be given during the talk. Some consequences will be also provided when considering some concrete operator ideals.

- Stéphane SEURET (Paris 12- Créteil)- modification since the version of May 14, 2015
  Random sampling of a Gibbs tree

  We perform a random sampling on a dyadic tree weighted by a Gibbs measure. We obtain a new multifractal object, with very interesting and non-classical properties. For instance, it exhibits (most of the time) two phase transitions. This has direct applications to the construction of new random wavelet series. This is a joint work with J. Barral.

See the pages at the address http://www.afo.ulg.ac.be/fb/meeting/genericity/?p=home

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